

# Percolating cluster of center vortices and confinement

Ferdinando Gliozzi<sup>a</sup>[torino]Dipartimento di Fisica Teorica, Università di Torino and INFN, sezione di Torino, via P. Giuria, 1, I-10125 Torino, Italy. E-mail: [gliozzi@to.infn.it](mailto:gliozzi@to.infn.it), [panero@to.infn.it](mailto:panero@to.infn.it), Marco Panero<sup>b\*</sup>, and Paolo Provero<sup>cd</sup>[torino]

<sup>a</sup>[

<sup>c</sup>Dipartimento di Scienze e Tecnologie Avanzate, Università del Piemonte Orientale “A. Avogadro”, and INFN, gruppo collegato di Alessandria, I-15100 Alessandria, Italy. E-mail: [provero@to.infn.it](mailto:provero@to.infn.it)

We study the role of percolating clusters of center vortices in configurations of an Ising gauge theory in 3D. It is known that low energy features of gauge theories can be described in terms of an “effective string picture”, and that confinement properties are associated with topologically non-trivial configurations. We focus our attention upon percolating clusters of center vortices, and present numerical evidence for the fact that these objects play a preminent role in confinement phenomenon, since their removal sweeps off confinement altogether. Moreover, numerical simulations show that the string fluctuations, and in particular the Lüscher term, are completely encoded in the percolating cluster.

## 1. INTRODUCTION

Center vortices are believed to play an important role in confining gauge theories: they disorder the gauge configurations and hence make the Wilson loop decay with the area law. This idea is quite old [1,2], but its investigation by lattice methods began recently [3,4].

We study a very simple, though non trivial, gauge theory, namely  $\mathbb{Z}_2$  gauge theory in three space–time dimensions: it turns out to be an ideal laboratory to test the relevance of center vortices to the confinement mechanism; in fact, in this theory, the gauge group coincides with its center, so the identification of center vortices can be performed without resorting to any gauge-fixing procedure. Moreover, the rather small size of configuration space enables us to get high precision numerical results from Monte Carlo simulations.

We focus our attention on the effect of the *size* of clusters of center vortices on confinement: the hypothesis we want to test, first proposed in Ref. [5], is that confinement in this model is due to the existence of an infinite cluster of center vortices,

i.e. a connected component, in the graph defined in the dual lattice by all the center vortices, whose size scales linearly with the lattice volume. Only such a giant component in the graph can disorder the gauge configurations enough to produce the area-law decay of the Wilson loop.

In order to shed light on the role of large clusters of center vortices, we first modify each configuration of a Monte Carlo-generated ensemble in the confined phase by removing the largest cluster of center vortices, and show that the ensemble thus obtained does not confine. On the other hand, if we remove all of the small clusters of center vortices, and leave the largest one only, confinement is preserved, albeit with a string tension significantly smaller than the original one. Remarkably, also the string corrections due to the quantum fluctuations of the confining flux tube are preserved by this transformation.

## 2. THE MODEL

The  $\mathbb{Z}_2$  gauge model is defined by the action

$$S(\beta) = -\beta \sum \sigma \quad , \quad \sigma = \prod_{\ell \in} \sigma_\ell \quad (1)$$

---

\*Presenter of the poster.

where the sum is extended to all plaquettes of a cubic lattice; the fundamental degrees of freedom are  $\mathbb{Z}_2$  variables  $\sigma$  defined on links  $\ell$ , and each plaquette contributes the product of its links to the action. The model enjoys a  $\mathbb{Z}_2$  gauge invariance: any local transformation consisting in flipping all of the  $\sigma$ 's defined on the links that meet at a given site, leaves the action invariant.

We can construct center vortices by assigning a vortex line in the dual lattice to each frustrated plaquette in the direct lattice. The resulting graph of center vortices in the dual lattice is gauge-invariant, and has an even coordination number.

Such graph is in general made of many connected components: the *clusters* of center vortices. The value of a Wilson loop in a given configuration is  $\pm 1$  according to the number, modulo 2, of vortex lines that are linked to the loop. A plausibility argument for the crucial role played by the infinite cluster of center vortices is the following (see also Ref. [5]): the only clusters contributing to the Wilson loop  $W(C)$  are those linked to  $C$ ; so, in the limit of large enough Wilson loops, clusters of finite size  $R$  can contribute to  $W(C)$  only when they are near  $C$ , and their number grows linearly with the length of the loop: therefore,  $\langle W(C) \rangle$  decays with a perimeter law, and the theory is deconfined. On the other hand, if there exists an infinite cluster of center vortices, then the number of vortex lines linked to the loop grows with its area, and the system is in the confining regime. It is easy to verify numerically that in the confining phase of  $\mathbb{Z}_2$  gauge system there is no ambiguity in finding a cluster of center vortices whose size scales linearly with the lattice volume for large enough lattices; conversely, in the deconfined phase the density of the largest cluster decreases rapidly with the volume.

### 3. ROLE OF THE LARGE CLUSTER OF CENTER VORTICES

To study the relation between the infinite cluster and the string tension value, we chose to simulate the model at  $\beta = 0.74883$ , which is well inside the scaling region, and for which the value of the string tension is known with high precision

[8,9]:  $\sigma = 0.01473(10)$ .

First, we verified that, in this confining regime, in the dual lattice configurations there exists a large cluster of center vortices, whose size grows linearly with the lattice volume.

Since VEV's of rectangular Wilson loops of sizes  $R \times T$  in the confining regime of a gauge theory are expected to behave as:

$$\langle W(R, T) \rangle = e^{-\sigma RT + p(R+T) + k} \left[ \frac{\eta(i\frac{T}{R})}{\sqrt{R}} \right]^{-\frac{d-2}{2}} \quad (2)$$

(where the term in square brackets is associated with quantum fluctuations of the colour flux tube), an efficient method [10] to extract the string tension  $\sigma$  from Wilson loop data generated by Monte Carlo simulations consists in considering loops of the same perimeter, but with different areas; we can define the following quantity:

$$\sigma_{\text{eff}}(L, n) \equiv \frac{1}{n^2} \log \left[ \frac{\langle W(L+n, L-n) \rangle}{\langle W(L, L) \rangle F(n/L)} \right] \quad (3)$$

where:

$$F(t) \equiv \left[ \frac{\eta(i)\sqrt{1-t}}{\eta(i\frac{1+t}{1-t})} \right]^{1/2} \quad (4)$$

The estimator  $\sigma_{\text{eff}}$  approaches the string tension for large  $L$ , when it is evaluated by data obtained from the Monte Carlo ensemble of configurations.

Now, if we erase the largest cluster of center vortices (in the dual lattice) in each of these configurations, the string tension estimator  $\sigma_{\text{eff}}$  approaches zero for large  $L$ : this means that removing the “infinite” cluster leaves us with non-confining configurations.

On the other hand, if we erase all clusters but the largest one, we still obtain confining configurations, but the string tension evaluated on these new configurations turns out to be significantly smaller than its value in the original Monte Carlo configurations. Another very interesting result (see [6,7] for details) is that the corrections due to the flux tube fluctuations survive the elimination of the small clusters.

These results may be compared with those obtained in other gauge theories [11,12].

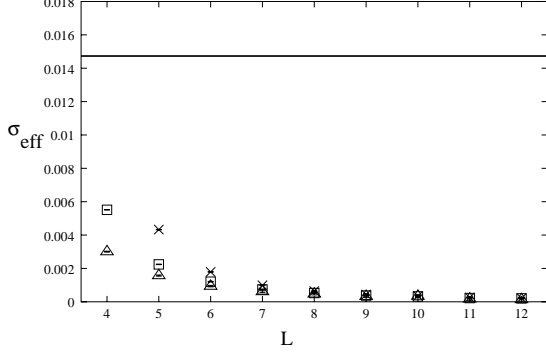


Figure 1. The string tension estimator evaluated after erasing the largest cluster in each configuration; different symbols refer to different values of  $n$ .

#### 4. CONCLUSIONS

Our numerical results show that:

- The largest center vortex in the dual lattice is responsible for confinement, but the string tension measured from configurations in which all other clusters have been removed does not reproduce the full string tension of the original theory: small clusters of vortices are unable by themselves to disorder the system enough to produce confinement, but they do give a finite contribution to the string tension of the full theory.
- Quantum fluctuations of the flux tube survive the elimination of the small clusters: after deletion of all the small clusters, the Wilson loop VEV shows the same shape dependence as in the full theory, which can be explained as originating by the fluctuations of a free bosonic string.

#### REFERENCES

1. G. 't Hooft, Nucl. Phys. B **138** (1978) 1.
2. G. Mack, in G. 't Hooft *et al.*, eds., *Recent Developments in Gauge Theories*, Plenum Press, NY.

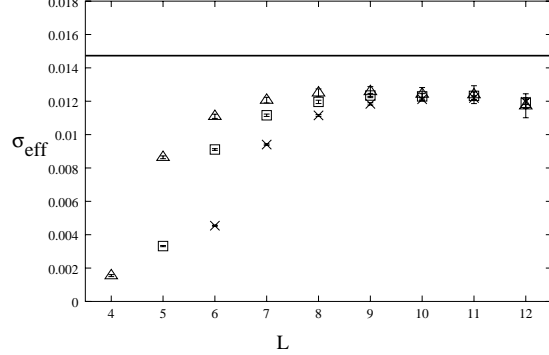


Figure 2. The string tension estimator evaluated after erasing all clusters but the largest one in each configuration. The solid line represents the string tension value in the original model.

3. L. Del Debbio, M. Faber, J. Greensite and S. Olejnik, Phys. Rev. D **55** (1997) 2298 [arXiv:hep-lat/9610005].
4. M. Faber, J. Greensite and S. Olejnik [arXiv:hep-lat/0204020].
5. M. Engelhardt, K. Langfeld, H. Reinhardt and O. Tennert, Phys. Rev. D **61** (2000) 054504 [arXiv:hep-lat/9904004].
6. F. Gliozzi, M. Panero and P. Provero, "Large center vortices and confinement in 3D  $\mathbb{Z}_2$  gauge theory", to be published in Phys. Rev. D [arXiv:hep-lat/0204030].
7. F. Gliozzi, M. Panero and P. Provero, contribution to the NATO Advanced Research Workshop "Confinement, Topology, and other Non-Perturbative Aspects of QCD", Jan. 21-27, 2002, Stara Lesna, Slovakia [arXiv:hep-lat/0205004].
8. M. Hasenbusch and K. Pinn, Physica A **192** (1993) 342 [arXiv:hep-lat/9209013].
9. M. Caselle, R. Fiore, F. Gliozzi, M. Hasenbusch, K. Pinn and S. Vinti, Nucl. Phys. B **432** (1994) 590 [arXiv:hep-lat/9407002].
10. M. Caselle, R. Fiore, F. Gliozzi, M. Hasenbusch and P. Provero, Nucl. Phys. B **486** (1997) 245 [arXiv:hep-lat/9609041].
11. A. Hart and M. Teper, Phys. Rev. D **58** (1998) 014504 [arXiv:hep-lat/9712003].

12. A. Hart and M. Teper, Phys. Rev. D **60** (1999) 114506 [arXiv:hep-lat/9902031].